GEOPHYSICS, VOL. 71, NO. 6 (NOVEMBER-DECEMBER 2006); P. F161–F164, 3 FIGS., 1 TABLE.
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A simple method of predicting S-wave velocity

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ABSTRACT

Prediction of shear-wave velocity plays an important role in seismic modeling, amplitude analysis with offset, and other exploration applications. This paper presents a method for predicting S-wave velocity from the P-wave velocity on the basis of the moduli of dry rock. Elastic velocities of water-saturated sediments at low frequencies can be predicted from the moduli of dry rock by using Gassmann's equation; hence, if the moduli of dry rock can be estimated from P-wave velocities, then S-wave velocities easily can be predicted from the moduli. Dry rock bulk modulus can be related to the shear modulus through a compaction constant. The numerical results indicate that the predicted S-wave velocities for consolidated and unconsolidated sediments agree well with measured velocities if differential pressure is greater than approximately 5 MPa. An advantage of this method is that there are no adjustable parameters to be chosen, such as the pore-aspect ratios required in some other methods. The predicted S-wave velocity depends only on the measured P-wave velocity and porosity.

INTRODUCTION

In some seismic applications, such as fluid substitution and amplitude variation with offset (AVO) analysis, estimating S-wave velocity, either theoretically or empirically, is essential. Castagna et al. (1985) present an empirical relation between S-wave velocities and P-wave velocities for water-saturated clastic silicates, which is known as the mudrock line. The mudrock line is useful for deriving S-wave velocities, but it underestimates S-wave velocity for unconsolidated sediments (Wang, 2000). Han et al. (1986) give a similar result. Wang (2000) introduces an empirical equation that predicts S-wave velocities using the bulk density of the saturated rock and the pore-fluid modulus as well as the P-wave velocity. Unlike the S-wave-velocity prediction by Castagna et al. (1985) and Han et al. (1986), Wang's empirical method is valid for fluids other than water in the pore space.

Methods for predicting S-wave velocities on the basis of rock physics also have been introduced. Greenberg and Castagna (1992) predict S-wave velocities from the Biot-Gassmann theory (BGT), under the assumptions that a robust relationship exists between P- and S-wave velocities and that nearly linear mixing laws for solid-rock constituents are valid. Xu and White (1996) predict S-wave velocity by using a combination of Kuster and Toksöz (1974) theory and the differential effective-medium theory, incorporating pore-aspect ratios to characterize the compliance of the sand and clay components. Jørstad et al. (1999) predicts S-wave velocities from the inclusion-based effective-medium theory and concludes that, whereas effective-medium theories are more complex than are the statistical regression methods, they have the advantage of incorporating the effect of clay and pore geometry directly in the formulation.

All of these methods that are based on rock physics require some parameters (e.g., pore-aspect ratio and differential pressure) to predict S-wave velocity from measured P-wave velocity and porosity. This paper presents a simple S-wave prediction method that uses only P-wave velocities and porosities of sediments and that is based on moduli of dry rock (Lee, 2005), under the assumption that velocity dispersion, attenuation, and anisotropy can be ignored. This theory is applied to velocities of consolidated sandstones as measured by Han et al. (1986) and to velocities for unconsolidated sediments, obtained from well logs acquired in the Alpine-1 well on the North Slope of Alaska. The S-wave velocities predicted from this theory agree well with measured S-wave velocities.

THEORY

Moduli of dry frame

Elastic velocities at low frequencies (i.e., P-wave velocity V_p and S-wave velocity V_s) of water-saturated sedimentary rocks can be computed from the Gassmann theory if the moduli of dry rock are known; however, within the poroelastic framework, dry moduli of the frame are undetermined and must be specified a priori. Generally, moduli are measured in the laboratory (Murphy et al., 1993), are

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predicted by theory [e.g., contact theory by Digby (1981)], or are derived under specific assumptions (Krief et al., 1990).

Pride et al. (2004) recommend the following dry-frame moduli for consolidated sandstones:

$$k_d = \frac{k_{ma}(1 - \phi)}{(1 + \alpha\phi)} \quad (1)$$

and

$$\mu_d = \frac{\mu_{ma}(1 - \phi)}{(1 + 1.5\alpha\phi)}, \quad (2)$$

where k_d and μ_d are bulk and shear moduli of the dry frame, respectively; k_{ma} and μ_{ma} are bulk and shear moduli of grains, respectively; and ϕ and α are consolidation parameters that represent porosity and the degree of consolidation between grains, respectively. Effective-medium theories can be approximately manipulated into expressions of this form, and they predict that α will depend both on the shape of the cavity and on the ratio μ_{ma}/k_{ma} (Pride et al., 2004). The factor 1.5 in equation 2 is somewhat arbitrary but yields an accurate V_p/V_s ratio for sandstones; factors 2 and 5/3 are also reasonable (Pride et al., 2004).

To generalize equation 2 (Pride et al., 2004), Lee (2005) proposes the shear modulus of a dry rock in the following equation:

$$\mu_d = \frac{\mu_{ma}(1 - \phi)}{(1 + \gamma\alpha\phi)}, \quad (3)$$

where

$$\gamma = \frac{1 + 2\alpha}{1 + \alpha}. \quad (4)$$

Note that when $\alpha = 1$, $\gamma = 1.5$, which is identical to equation 2. When $\alpha = 2$, $\gamma = 5/3$, and as α increases, γ approaches 2; therefore, equation 4 covers all reasonable values that were suggested by Pride et al. (2004). Using the moduli of a dry frame shown in equations 1 and 3, velocities of water-saturated rocks at low frequencies can be calculated by using the Gassmann theory and assuming that pore fluid does not change the shear modulus.

Predicting S-wave velocity

Equations 1 and 3 can be used to predict S-wave velocities from P-wave velocities and porosities of water-saturated sandstones because one parameter α relates both bulk and shear moduli of a dry frame. Defining the predicted P-wave velocity using the BGT, with the dry moduli derived from equations 1 and 3 as V_p^* and as V_p^m for measured P-wave velocity, the consolidation parameter can be calculated by solving the following equation:

$$V_p^*(\alpha) - V_p^m = 0. \quad (5)$$

Therefore, the shear modulus can be calculated using equation 3 by substituting the consolidation parameter that is estimated from equation 5 into equations 3 and 4. The S-wave velocities can be calculated from $V_s = \sqrt{\mu_d/\rho}$, where ρ is the bulk density of the sediment.

Figure 1 shows consolidation parameters calculated by solving equation 5 with the elastic constants shown in Table 1, using the Newton-Raphson method (Press et al., 1986) and the predicted S-wave velocities for sandstones that were measured at 5 and 40 MPa by Han et al. (1986). The bulk and shear moduli of composite grains, including clay, are calculated using Hill's (1952) averaging method. The average consolidation parameter for 5-MPa data is 5.4 and for 40-MPa data is 3.2. The fractional errors of predicted S-wave velocities are 0.01 ± 0.04 for 5-MPa data and 0.00 ± 0.04 for 40-MPa data.

Figure 2 shows input data for predicting S-wave velocities for unconsolidated sediments at the Alpine-1 well, North Slope of Alaska, and Figure 3 shows the predicted S-wave velocities. The fractional error of S-wave velocities when using the proposed method is

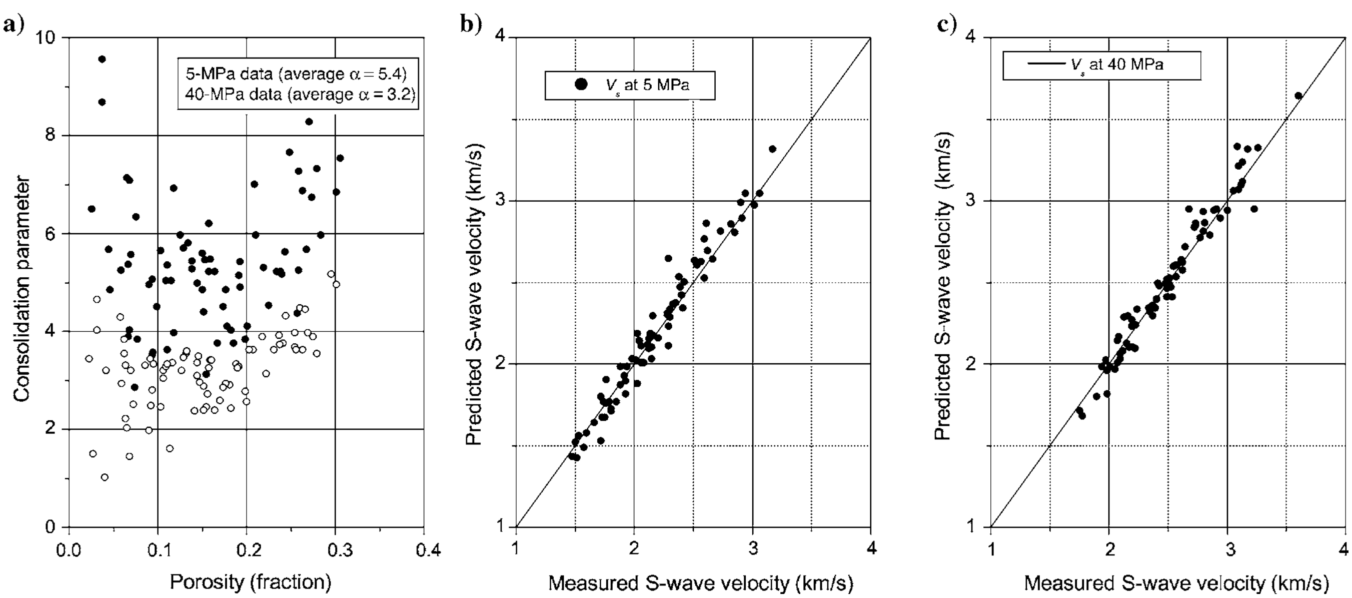

Figure 1. Comparison of predicted S-wave velocities, which are calculated from the P-wave velocities and porosities using the dry-rock moduli from equations 1 and 3, and measured S-wave velocities for sandstones obtained by Han et al. (1986): (a) Calculated consolidation parameters for 5- and 40-MPa data. (b) Velocities measured at 5 MPa. (c) Velocities measured at 40 MPa.

Table 1. Elastic constants used for this study (from Lee, 2005).

	Values used
Shear modulus of quartz	44 GPa
Bulk modulus of quartz	38 GPa
Shear modulus of clay	6.85 GPa
Bulk modulus of clay	20.9 GPa
Bulk modulus of water	2.29 GPa
Density of quartz	2650 kg/m^3
Density of clay	2580 kg/m^3

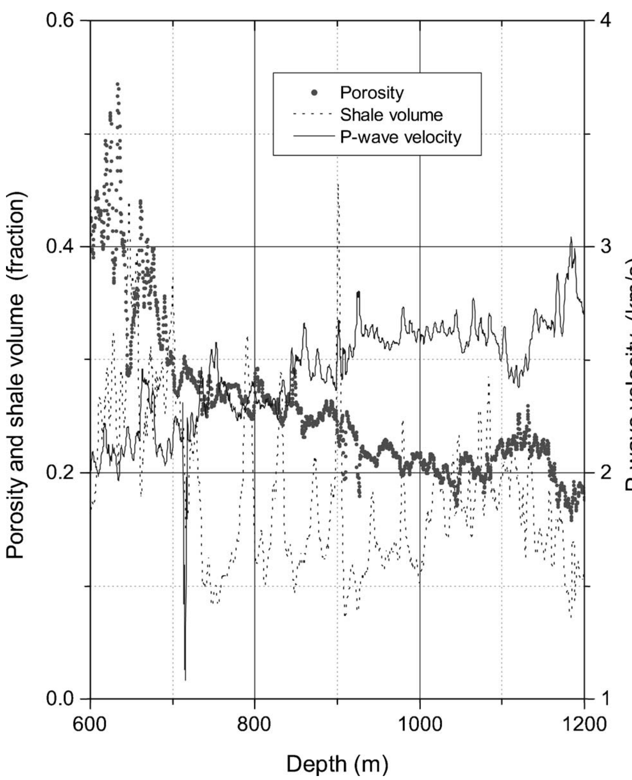

Figure 2. A graph showing input data used for the S-wave prediction (P-wave velocity with a solid line, density porosity with dots, and shale volume calculated from a gamma log with a dotted line) at the Alpine-1 well, North Slope of Alaska.

-0.02 ± 0.10, whereas it is 0.03 ± 0.10 when using the Pride et al. (2004) method and it is 0.01 ± 0.12 when using the least-squares fitting (LSF) method given by $V_s = -0.570 + 0.658V_p$. Although the S-wave velocity predicted by using the LSF method calibrated at the Alpine-1 well is the most accurate, it is not a practical prediction method because both P-wave data and S-wave data are required to estimate the LSF coefficients. Figure 3 indicates that the accuracy of the method proposed in this paper is comparable to that of the LSF method, and that this method also works well for S-wave velocity less than 1 km/s. Using equation 4 instead of a constant of 1.5 as proposed by Pride et al. (2004) improves the accuracy of predicted S-wave velocity.

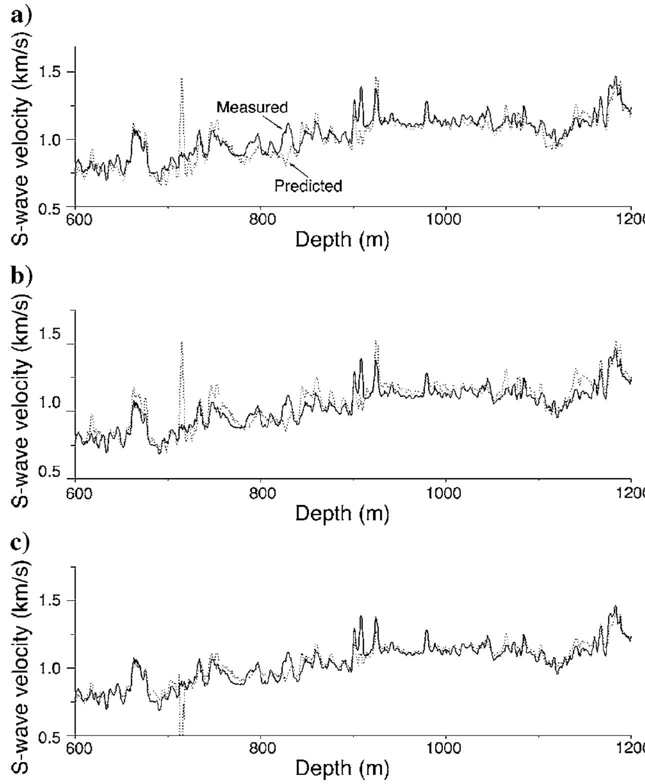

Figure 3. Comparison of predicted S-wave velocities from the P-wave velocities and porosities at the Alpine-1 well, North Slope of Alaska, using various methods. (a) Using the proposed equation. (b) Using the moduli proposed by Pride et al. (2004). (c) Using an LSF equation, $V_S = -0.570 + 0.658V_P$, derived at the Alpine-1 well.

The consolidation parameter decreases as differential pressure increases, porosity decreases, and the degree of consolidation increases (Lee, 2005). As the degree of consolidation of sediments decreases (sediments become unconsolidated), the consolidation parameter increases (Lee, 2005). The calculated consolidation parameters for the Alpine-1 well range from 8 to 40, and average 22. Although the consolidation parameter is defined for consolidated rocks (Pride et al., 2004), the results indicate that the same formulation can be applied to unconsolidated sediments by using larger consolidation parameters.

The proposed theory was tested for sediments under differential pressures greater than 5 MPa and burial depths greater than 600 m; therefore, whether the proposed method is appropriate for sediments at very shallow depths remains to be determined. This method also assumes that the Gassmann equation is valid for water-saturated sediments; therefore, the basic assumptions for the Gassmann equation, e.g., that a porous medium is macroscopically homogeneous and isotropic, are applicable to this method.

CONCLUSIONS

Elastic velocities of water-saturated sediments at low frequencies can be predicted from the moduli of dry rock by using Gassmann's equation. If the moduli of dry rock can be estimated from P-wave velocities, then S-wave velocities easily can be predicted from the moduli. There are equations for the moduli of dry rock that relate the bulk modulus of sediment to the shear modulus through a compaction constant; therefore, the shear modulus can be calculated using the P-wave velocities by solving a coupled equation. Numerical results indicate the predicted S-wave velocities for consolidated and unconsolidated sediments agree well with measured velocities. An advantage of this method is that there are no adjustable parameters to be chosen in the formulation, so the predicted S-wave velocity depends solely on the measured P-wave velocity and porosity.

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