Supporting information about the fit for ultra-high densities

For a fit over a large density range it is convenient to use reduced units determined from the critical wave vector $q_c$ for Landau damping. Wave vectors are expressed as $q = \tilde{q}/q_c$ and frequencies are then expressed in the critical frequency $\omega_c \equiv \omega_{pl}(q_c)$, easily found from the upper single particle band edge as $\hbar(q_c^2 + 2q_c k_F)/2m^*$. The following approximation for the turned out to describe the plasmon dispersion excellently

$$\tilde{\omega}^{\text{fit}}_{pl}(q, r_s) = \frac{\omega^{\text{fit}}_{pl}(q, r_s)}{\omega_{pl}(q, r_s)} = \sqrt{q} P_{2,4}(\tilde{q}, r_s), \quad (S.1)$$

which can be expanded to

$$\tilde{\omega}^{\text{fit}}_{pl} = \sqrt{1 + \frac{p_0 + p_1 \tilde{q} + p_2 \tilde{q}^2}{1 + \tilde{p}_1 q + \tilde{p}_2 q^2 + \tilde{p}_3 q^3 + \tilde{p}_4 q^4}} \quad (S.2)$$

Next, these coefficients were fitted in a sectioned step to the density parameter $r_s$ via the ansatz:

$$p_i = \pm c_0^{(i)} \pm c_1^{(i)} r_s \pm (c_2^{(i)} r_s)^{3/2} \pm (c_2^{(i)} r_s)^2 \quad (S.3a)$$

$$\tilde{p}_i = \pm \tilde{c}_0^{(i)} \pm \tilde{c}_1^{(i)} r_s \pm (\tilde{c}_2^{(i)} r_s)^{3/2} \pm (\tilde{c}_2^{(i)} r_s)^2 \quad (S.3b)$$

The coefficients can be nailed down according to the limits in Eq. (D.1) and Eq. (D.2) that were calculated as a first step. Using the expression (25) found for $q_c(r_s)$, this procedure gives the following relations for the coefficients up to $O(q^3)$

$$p_0 = 2^{3/4} \sqrt{r_s} \frac{\sqrt{q_c/k_F}}{\hbar \omega_c/\varepsilon_F} = 2^{3/2} \frac{\sqrt{q_c q_h}}{r_s \hbar \omega_c [\text{Ry}^*]} \quad (S.4)$$

$$p_1 - p_0 \tilde{p}_1 = \frac{1 + n \kappa(r_s) \varepsilon_F}{2^{7/4} \sqrt{r_s}} \frac{(q_c/k_F)^{3/2}}{\hbar \omega_c/\varepsilon_F} = \frac{1 + \kappa(r_s)/\kappa^0(r_s)}{2^{3/2} r_s} \frac{(q_c a_{0*})^{3/2}}{\hbar \omega_c [\text{Ry}^*]} \quad (S.5)$$

Here, the non-interacting compressibility in 2D is $\kappa^0 = 1/n \varepsilon_F$. The values of the remaining parameters $p_i, \tilde{p}_j$ were computed numerically.

To achieve the best results, a final fitting loop incorporating aspects of generic algorithms (e.g. mutation and crossover) was applied. As objective function, a weighted least-square-error was used. This resulted in the coefficients given in Tables D.2 and S.1. In figure 3, the final approximant is compared to the numerical data with the former values; the parameters of table S.1 are superior when $r_s \gtrsim 30$ and correct within 5% down to $r_s \gtrsim 10$. Note that conversion from reduced to real units introduces a density dependent scaling, as $\varepsilon_F \propto r_s^{-2}$, having a strong impact for dense systems. As there the RPA gives results of sufficient accuracy, the fit proposed here is designed to capture the wide density range of intermediate $r_s$ up the predicted Wigner crystallization.
where these coefficients were obtained by fitting the actual 2p2h peak to a Lorentzian. Alternatively, one can start with the expression for \( \chi^{2p2h}(q,\omega) \) given in the main text in Eqs. (16)-(22). Written as a single numerator \( N(q,\omega) \) and denominator \( D(q,\omega) \) this is then Taylor expanded around \( \omega_{pl}(q) \)

\[
\begin{align*}
\Gamma_{2p2h}^{fit} & = q^{7/2} \left( p_0^{\Gamma} + p_{1/2}^{\Gamma} q^{1/2} + p_{3/2}^{\Gamma} q^{3/2} + p_{4/2}^{\Gamma} q^{4/2} \right), \quad q = q/q_c \\
p_{i/2}(r_s) & = c_{i/2,0}^{p} + c_{i/2,1}^{p} r_s + c_{i/2,2}^{p} r_s^2 ,
\end{align*}
\tag{S.6a}
\tag{S.6b}
\]

with the coefficients given in Table S.2.

These coefficients were obtained by fitting the actual 2p2h peak to a Lorentzian. Alternatively, one can start with the expression for \( \chi^{2p2h}(q,\omega) \) given in the main text in Eqs. (16)-(22). Written as a single numerator \( N(q,\omega) \) and denominator \( D(q,\omega) \) this is then Taylor expanded around \( \omega_{pl}(q) \)

\[
\begin{align*}
-\Im \chi^{2p2h}(q,\omega) & = -\Im \frac{N(q,\omega)}{D(q,\omega)} \approx \frac{\Re N(q,\omega) \Im D(q,\omega)/c(q)}{(\omega_{pl}(q) - \omega)^2 + (\Im D(q,\omega)/c(q))^2} ,
\end{align*}
\tag{S.7}
\]

where \( c(q) \equiv \partial \Re N/\partial \omega|_{\omega_{pl}} \) and \( \Im N \Re D \ll \Re N \Im D \). All functions contributing to \( N \) and \( D \) being available in the full implementation of \( \chi^{2p2h} \), we confirmed that the Lorentzian (S.7) (blue full curve in Fig. 4) closely matches that obtained with (S.6) (green dotted curve).

<table>
<thead>
<tr>
<th>( p_{i/2}(r_s) )</th>
<th>( c_{i/2,0}^{p} )</th>
<th>( c_{i/2,1}^{p} )</th>
<th>( c_{i/2,2}^{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_0^{p}(r_s) )</td>
<td>40.0298</td>
<td>-6.9251</td>
<td>0.315122</td>
</tr>
<tr>
<td>( p_{1/2}^{p}(r_s) )</td>
<td>-94.6375</td>
<td>16.0834</td>
<td>-0.681113</td>
</tr>
<tr>
<td>( p_{3/2}^{p}(r_s) )</td>
<td>129.301</td>
<td>-21.1785</td>
<td>0.803636</td>
</tr>
<tr>
<td>( p_{4/2}^{p}(r_s) )</td>
<td>-74.6926</td>
<td>12.0368</td>
<td>-0.437983</td>
</tr>
</tbody>
</table>

Table S.2. Coefficients for the fit of the dispersion of the sheet plasmon used in (S.6).
Figure S.1. Plasmon half-width $\Gamma$ due to 2-pair excitations (lines) compared with measured values (symbols). Left: dense case (Nagao et al. [17], $r_S = 1.2$ and Rugeramigabo et al. [18], $r_S = 1.0$). Right: highly dilute case (Hirjibehedin et al. [19, 20], $r_S \approx 9...20$). In units of Fermi energies and wavevectors, higher lying curves correspond to higher $r_S$.

In Fig. S.1 we give the comparison with actually measured half widths $\Gamma$, proportional to a plasmon’s life time $\tau^{-1}$. As also the case in the bulk, plasmon decay into multi–pair excitations is not possible at $q=0$ and experimental $\Gamma(0)$ are due to other mechanisms. For dense 2DEGs, where multi-pair excitations get negligible, plasmon losses by pair excitations are more than 2 orders of magnitude below the measured data. For the ultra-dilute samples the 2p2h damping is still an order of magnitude too low.

Certainly, not only the magnitude, but also the overall shape of the 2p2h width-dispersion does not satisfactorily match the experimental data. The decrease of $\Gamma(q)$ when it approaches the single–particle band is expected to be overcome by higher order multi-pair effects which get relevant with increasing $q$ and $r_S$.

References


